

Note we shall use the symbol P for a ~~projector operator~~ ^{projector} to denote unambiguously

(1) The projector P

(2) The observable P associated with the projector operator $\hat{P}$

(3) The subspace of Hilbert space which is the range of $\hat{P}$

(4) The projector: $[P] = 1$

(5) The proposition: The state ~~lies in~~ ^{the state vector lies in the range of} $\hat{P}$

(6) Moreover if P is associated with the total set Δ and the projector-valued measures associated with the observables then we use P to denote

(6) The proposition: $[Q] \in \Delta$

The senses (4), (5) are equivalent under the assumption of FVNC

$$[F(Q)] = f([Q])$$

The senses (1) and (5) should be clear from the context.

Putnam on Quantum Mechanics:

Logic, Quantifier and the ^{Two} 2-Slit Experiment.

In Putnam's (1989) treatment of the 2-slit experiment he employs ~~the~~ ^{the} lattice-theoretic (LT) version of quantum logic⁽⁸²⁾ in which realizations in terms of Boolean algebras were replaced by realizations in terms of the projection lattice of a Hilbert space.

Adapting a Carnapian probabilistic theory definitions of conditional probabilities Putnam argued as follows:

of R_i is the proposition that an electron hits region Δ_i on the screen so that R_i is associated with the projection operator $P_i = \int_{\Delta_i} dP(x)$, where

^{generates}
 $P(x)$ is the projection-valued measure associated with the position observable (i.e. $x = \int x dP(x)$ in one-dimension)

and similarly let A_1 and A_2 correspond to projectors onto the two ^{subspaces} of states ~~it is~~ ^{that} associated with passage through one or other slit

Then Putnam writes

$$\text{Prob}(R_i | A, \vee A_2) = \frac{\text{Prob}(R_i \wedge (A, \vee A_2))}{\text{Prob}(A, \vee A_2)} \quad (1)$$

^{of} distributivity law now valid

$$\begin{aligned} \text{Prob}(R_i \wedge (A, \vee A_2)) &= \text{Prob}((R_i \wedge A) \vee (R_i \wedge A_2)) \\ &= \text{Prob}(R_i \wedge A) + \text{Prob}(R_i \wedge A_2) \end{aligned}$$

$$= \text{Prob}(B_i | A_1) \text{Prob}(A_1) + \text{Prob}(B_i | A_2) \text{Prob}(A_2) \quad \text{--- (3)}$$

Then Taking $\text{Prob}(A_1 \vee A_2)$

$$\text{Prob}(A_1) = \text{Prob}(A_2) = \frac{1}{2} \text{Prob}(A_1 \vee A_2) \quad \text{--- (4)}$$

gives

$$\begin{aligned} \text{Prob}(B_i | A_1 \vee A_2) \\ = \frac{1}{2} \text{Prob}(B_i | A_1) + \frac{1}{2} \text{Prob}(B_i | A_2) \end{aligned} \quad \text{--- (5)}$$

This is the classical pattern with no interference.

To check the Copenhagen Subman. denied the disjunctive law.

But Gerdner (1971) argued that in this particular case the disjunctive law is trivially valid, since

$$\left. \begin{aligned} P_i \wedge P_{\Psi_1} &= 0 \\ P_i \wedge P_{\Psi_2} &= 0 \\ P_i \wedge (P_{\Psi_1} + P_{\Psi_2}) &= 0 \end{aligned} \right\} \quad \text{--- (6)}$$

(Essentially what Gerdner is pointing out is that states picked out by P_i are never in the linear span of Ψ_1 or Ψ_2)

A rigorous proof of this result was provided by Gribens and Pearson (1981)

So Putnam's attempt failed. But even
 up to now blocked the classical
 distribution, there was ~~nothing~~ ^{the specific pattern of} ~~any~~ ^{quantum-mechanical}
 the argument to demonstrate interference.

Now is the 2nd approach to DL,
 incompatible properties can be
 combined ~~so as~~ to produce logical
 contradictions, and this was regarded
 by Putnam as an important feature
 of his DL account.

But, in an argument first published
 in his (1981) Putnam claimed to
 establish that no. could prove
 incompatible properties although not
 use them for simultaneous predictions.

So incompatible properties do
 not correspond to contradictions,
 although ~~they~~ cannot be asserted as a
 single bit. This leads back
 to a motivation for the Paradox-Boden-
 aligned version of DL of which was
 introduced originally by S. Koster and
 Speiser in their (1967). Here
 incompatible properties cannot be
 conjoined, so statements like

$R \vdash \neg (A_1 \vee A_2)$ are not well-formed,

and a quite different approach is
 required for the DL analysis
 of conditional probability.

This was provided by Friedman and
 Putnam (1978) who tackle conditional

*

Question Friedman and Putnam show that

(8) justify eq (7) by substituting

$P_1, P_2 \in \mathcal{P}$ in the left of
proving P_1 , and then applying the
rule (7) for maximal reducts.

probabilities directly rather than by
defining them in terms of joint
distributions.

So for a maximal measurement
establishing the value of F - a

1-dimensional projector P_F they write

$$\text{Prob}_\phi(F | P) = \frac{\text{Prob}_\phi(F, P_\phi)}{\text{Prob}_\phi(F | P_\phi)} \quad (7)$$

where ϕ is the initial state and P_ϕ is an arbitrary projector (projector)

Now they take the case of *coherization*
on a 2-dimensional projector $P_1 + P_2$
where for the 2-part case, P_1 corresponds
to P_{Ψ_1} and P_2 to P_{Ψ_2} .

To get the right QM interpretation what
they need - a Lüders rule that would
give

$$\text{Prob}_\phi(R_i | P_1 + P_2) = \frac{\text{Prob}_{(P_1 + P_2)\phi}(R_i)}{\| (P_1 + P_2)\phi \|^2} \quad (7)$$

where we use the convention that any state is
understood to be normalized, so

$$(P_1 + P_2)\phi \text{ stands for } \frac{(P_1 + P_2)\phi}{\| (P_1 + P_2)\phi \|}$$

To justify (7), Friedman & Putnam note that

$$P_\phi \Rightarrow (\checkmark (P_1 + P_2)\phi \sim P_1 + P_2) \quad (8)$$

where $\Rightarrow$ signifies semantic (logical) consequence
and $\sim$ denotes mutual equivalence.

* Question

* Question So the justification for
identifying the Fregean - Putnam
construction with a generalized modal
superspace remains contentious

But ~~DL~~ Friedman and Putnam define 2
ways a dissolved form of etc
classical definition with DL
consequences

3

$$\text{So } A \sim B \stackrel{\text{def.}}{=} (A \wedge B) \vee (\neg A \wedge \neg B) \quad (9)$$

So classical logic the RHS of (9)
is equal to

$$(A \supset B) \wedge (B \supset A)$$

where $\supset$ is material implication

$$\text{i.e. } A \supset B = \neg A \vee B \quad \dots (10)$$

It was shown by Hellmann (1981)

that Putnam's Friedman-Putnam
definition of natural equivalence can be
identified as

$$A \sim B = \models (A \supset_s B) \wedge (B \supset_s A) \quad (11)$$

where $\supset_s$ is the Sasaki look

$$A \supset_s B = \neg A \vee (A \wedge B) \quad \dots (12)$$

which reduces to the classical definition if
we are allowed to distribute.

But in DL the Sasaki look fails, e.g., to
satisfy the deduction theorem

$$A \Rightarrow B \text{ iff. } A \supset_s B \text{ is a tautology.}$$

*

Be Oct as it may, conditionalization — really involves a subordinate condition with a probabilistic consequent.

So we could test $\text{Prob}(A|B) = p$ as

$$B \square \rightarrow (\text{Prob}(A) = p). \quad (13)$$

we ask the question:

10. If B were the case, what would the probability of A be?

We now raise

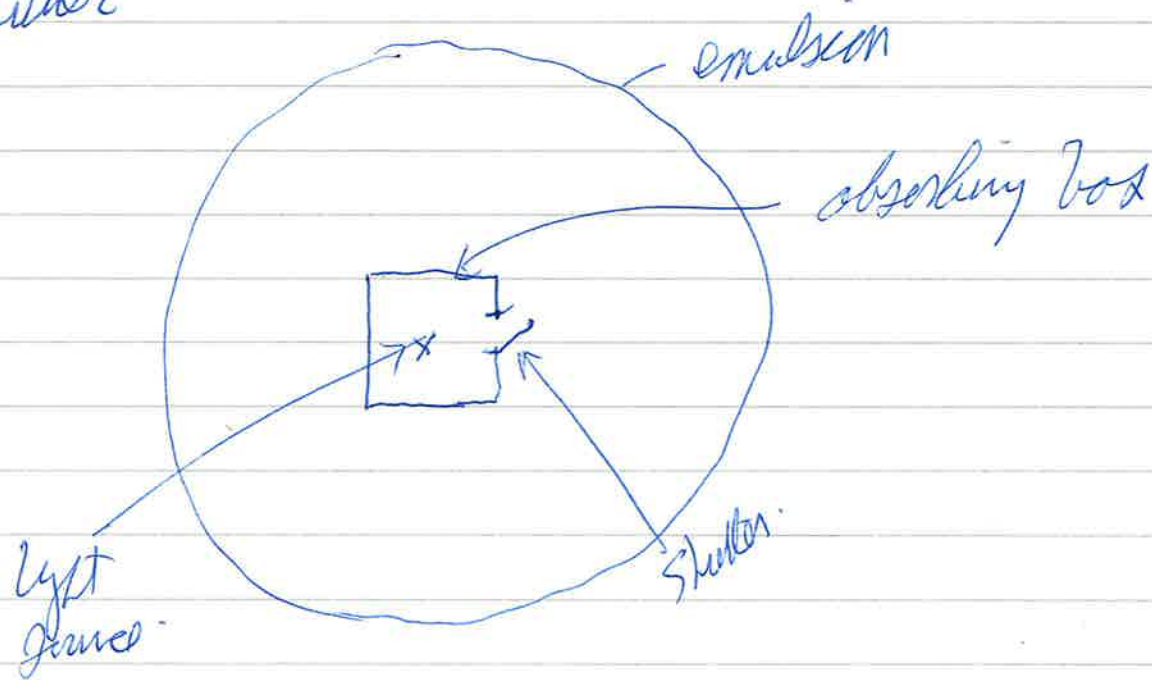
~~the~~ question of whether we can substitute materially equivalent propositions for B in ex. (13)

The answer is clearly no, since any two false propositions, no matter how equivalent, and the result in the classical case would give us pause in treating the test case.

Friedman and Putnam have been criticized by Hellman, But not Stairs for their treatment of conditionalization, albeit for rather different reasons.
 Their own critique is directed to Hellman.

But I now want to criticize Putnam's move to the PBA approach motivated by his (1981) treatment of simultaneous propositions of incompatible events.

Rubnam designs a thought experiment involving the escape of a photon through a shutter in the wall of an ~~absorbing~~ ^{absorbing} box which is then detected by a spherical photographic emulsion unit, its centre located at the shutter.



At the time t at which the photon reaches the detector, Rubnam writes for the DM state,

$$\bar{\Psi} = \frac{1}{\sqrt{2}} (\bar{\Psi}_{in} + \bar{\Psi}_{out}) \quad (14)$$

where $\bar{\Psi}_{in}$ is a state confined to the interior of the box and $\bar{\Psi}_{out}$ a state confined to the exterior of the box.

Suppose we divide the whole region of space into discrete cells Δ_i and define projection operators associated with these cells as before, P_i^{in} if Δ_i lies inside the box or P_i^{out} if Δ_i lies outside the box.

Then clearly:

$$\Psi_{out} = \left(\sum_i P_i^{out} \right) \Psi \quad (15)$$

$$\text{and } \Psi_{in} = \left(\sum_i P_i^{in} \right) \Psi \quad (16)$$

where the summation in (15) is over all cells located outside the box and the summation in (16) is over all cells located inside the box. and we have again used the convention that all states are to be specified up to normalization.

We also have - results like

$$\sum_i \{ P_i^{in} \} + \sum_i P_i^{out} = 1 \quad (17)$$

$$\begin{aligned} \text{and } P_i^{out} &\Rightarrow \sum_i P_i^{out} \Rightarrow \sum_i P_i^{in} + \sum_i P_i^{out} \\ &\Rightarrow \sum_i P_i^{in} + \sum_i P_i^{out} \end{aligned} \quad (18)$$

Now suppose we measure P_R^{out} where P_R encodes a small region of the envelope and find the value 1, i.e. the photon has been recorded as measured recorded as going a bit in P_R , then, says Patnam we know P_R^{out} and also P_{Ψ}^{out} , which clearly do not commute. Patnam's argument is that any physicist would interpret finding a photon in P_R as

imposed that a photon had passed through the shutter which Putnam then proceeds to identify with P_{Ψ}^{out} having the value one.
But this is ~~allowing~~ ^{is} ~~conflation~~!

From (18), identifying i with h we have
$$P_h^{out} \Rightarrow \sum_i P_i^{out}$$

So in words, if the photon is somewhere specifically outside the box, i.e. in D_A , then it is also somewhere outside the box.

But P_h^{out} commutes with $\sum_i P_i^{out}$, so there is no question of proving incompatible propositions in virtue of proving both that the photon is in D_A and it is somewhere outside the box.

Note also that $P_{\Psi}^{out} \Rightarrow \sum_i P_i^{out}$

so if we knew that the state of the photon was Ψ^{out} then we would know it was outside the box. But the implication does not go the other way.

Essentially Putnam's mistake boils down to claiming that

$$P_h^{out} \Rightarrow P_{\Psi}^{out}$$

apparent

in the mistaken assumption that one can

Reaction

*

Of course Putnam could respond
by making the (1978) move, that

$$P_{\Phi} \Rightarrow (P(\sum_i P_i^{out}) \Phi \Leftrightarrow \sum_i P_i^{out}) \quad (20)$$

10. From the fact that the state is
 Φ allows us to infer the
material equivalence of the ^{two} incompatible
propositions $P(\sum_i P_i^{out}) \Phi$ and $\sum_i P_i^{out}$

But we have already seen fit to
criticise the cogency of this argument,

Furthermore Putnam does not cite
this argument in support of his
(1981) claim, and my reading of
the (1981) is that it is supposed to
stand on its own feet, without
relying on ~~the~~ potentially compromised
inferences of (20).

equation $P_{\Psi_{out}} = \sum_i P_i^{out} \Psi$

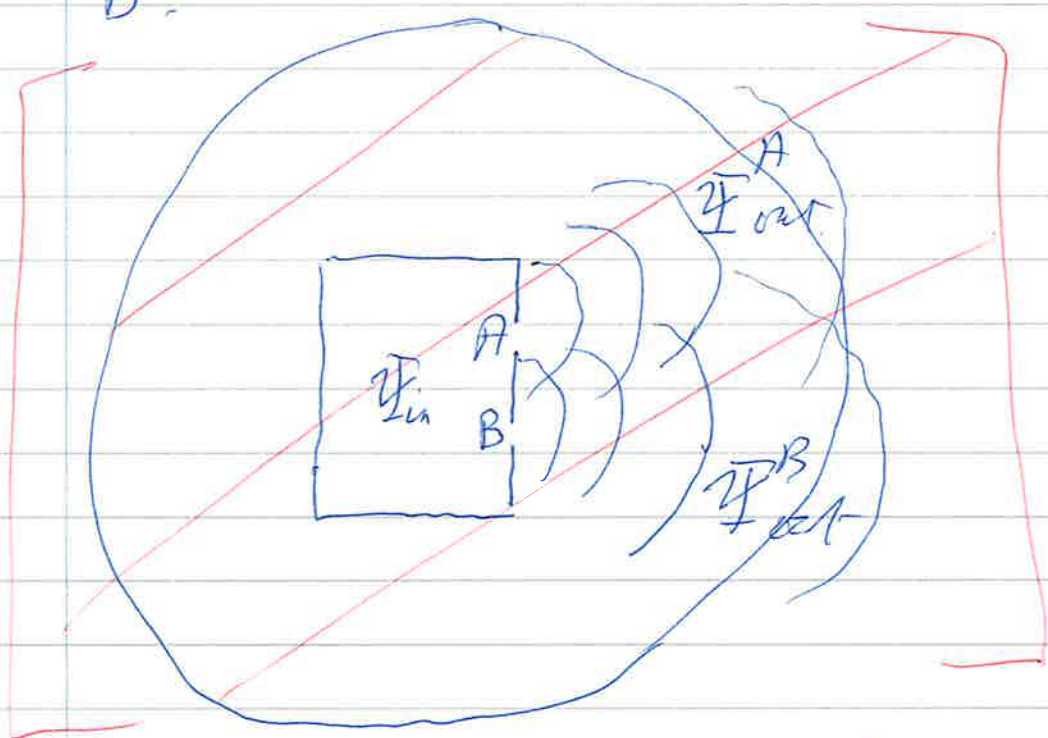
with $\sum_i P_i^{out} = 1$

Quantum

For the sake of completeness let us run through the analysis used there are two shutters in the box, so making the comparison with the two-slit experiment. Call the shutters A and B and write, in analogy with Equation (14),

$$\Psi = \frac{1}{\sqrt{2}} \Psi_{in} + \frac{1}{2} \Psi_{out}^A + \frac{1}{2} \Psi_{out}^B$$

where Ψ_{out}^A is the state at time t of the photon passing through shutter A and Ψ_{out}^B for the photon passing through B. - (19)



Again suppose the photon hits at D_B
 so we have $P_B = 1$.
 Again this contradicts the fact that the photon

is outside the box, as before represented
by the entirely compatible proposition
 $\sum_i P_i^{\text{out}}$.

But this does not enable us to know
what sort the photon went through.

We cannot infer the truth value for

$P_{\Psi_{\text{out}}^A + \Psi_{\text{out}}^B}$, let alone the truth
values for $P_{\Psi_{\text{out}}^A}$ or $P_{\Psi_{\text{out}}^B}$ separately.

The conclusion of this argument is
that Putnam has ^{done} nothing to
convince us that we ^{can} have incompatible
properties, so the motivation for
moving to PBA as compared with LT
versions of QL is ~~lost~~. cannot be
sustained.

Putnam claims (p. 258 of his (1981)) that
his thought experiment "clears up a
problem which hung me up for nearly
twenty years" I fear that the
problem still remains!

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11

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13 ~~12~~ PUTNAM, H. (1981):

8 PUTNAM, H. (1957):

9 PUTNAM, H. (1965a):

10 PUTNAM, H. (1965b):

③ FRIEDMAN, M. and PUTNAM, H. (1978):

④ ~~5~~ GARDINER, M. (1971):

12 ~~13~~ PUTNAM, H. (1974):

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14/ REDHEAD, M. L. G. (1987): Incompleteness,
nonlocality, and Realism: A Prolegomenon to the
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(1) Burkhoff, G. and von Neumann, J. (1936)

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Putnam's papers on QM

1957 3-valued logic

Third truth value $\perp$ for which
all parties want tho' in 2-val
exp.

1965 A philosopher looks at Quantum
Mechanics

1969 Is logic Empirical?

1974 How to think quantum-mechanics?

1978 (joint with R. Friedman)
Quantum logic, Conditional probability
and Inference (Dialectica)

1981
(1988) Quantum Mechanics and the Observer
(Erkenntnis)

Logic, Quanta, and the 2-Slit Experiment

New Transparencies (a) 6 Series of P
(b) Convention for $\delta\phi \rightarrow \frac{\delta\phi}{\delta\phi_1}$

1. Putnam & all 2-slit Experiment

2. contd

3. Friedman & Putnam (1978)

4. Approx Material equilibrium cont.

5. Correspondence

6. Putnam Right Experiment (1981)

7. contd.

8. Flows in Putnam's Argument

9. The 2-slit version of Putnam's argument

9. same contd.

Old Transparencies Show

1 Inputs to QL

3 & 4 Historical Development of QL

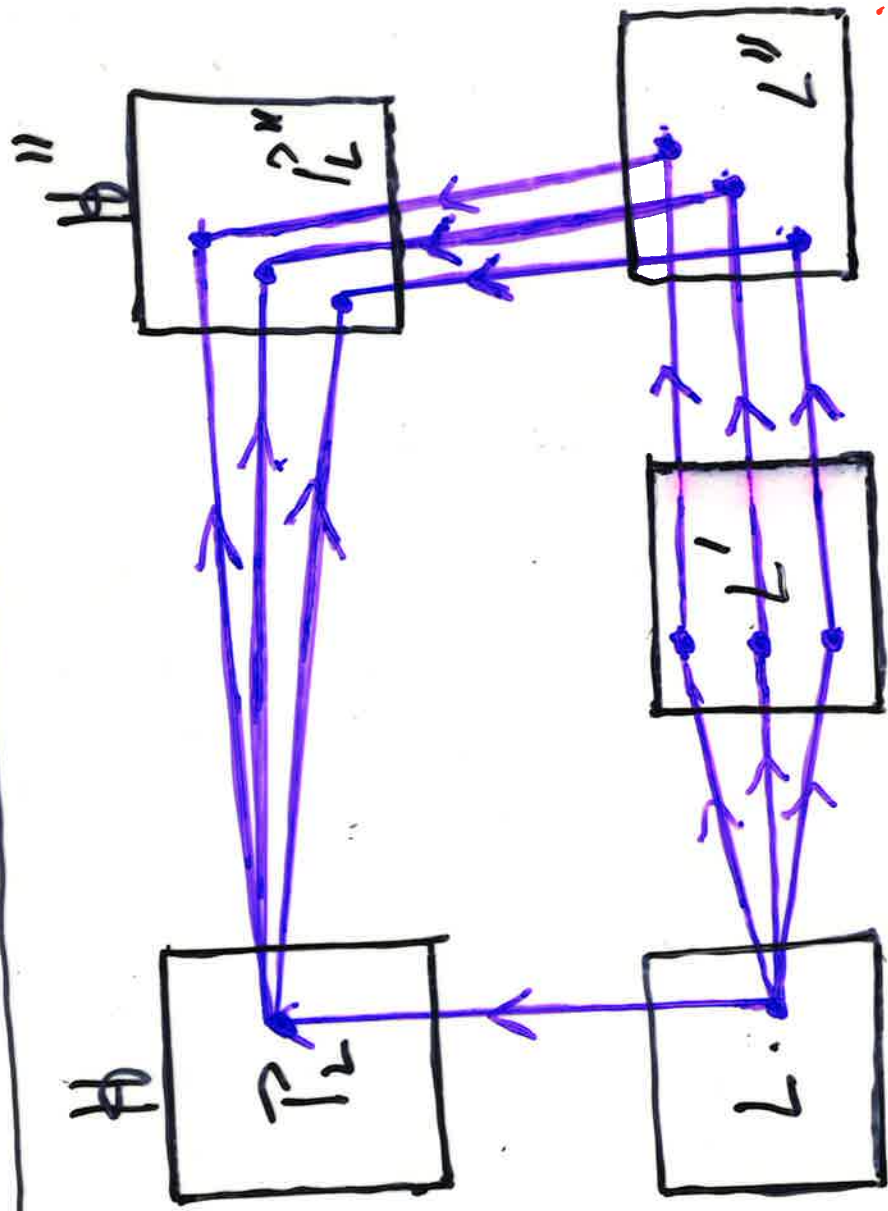
14. Putnam & Q.L.

15. Putnam & the Paradoxes

18. Quantum States & Putnam's Idea

10a. Admissible truth valuations for QL

PUTNAM STATES



Propositions
about QM
states

Propositions
about values
of observables

Propositions
about
Putnam
states

(1a)

Note we shall use the symbol $\hat{P}$ for a projector to denote unambiguously:

- (1) The projection operator $\hat{P}$
- (2) The observable $\hat{P}$ associated with the operator $\hat{P}$
- (3) The subspace of Hilbert space which is the range of $\hat{P}$

(4) The proposition: $[\hat{P}] = 1$

(5) The proposition: The state vector $\hat{P}$ lies in the range of $\hat{P}$

Moreover, if $\hat{P}$ is associated with the Borel set Δ via the projection-valued measure associated with observable $\hat{Q}$

(6) The proposition: $[\hat{Q}] \in \Delta$

(4) and (6) are equivalent if we assume FUNC:
 $[\hat{P} + \hat{Q}] = [\hat{P}] + [\hat{Q}]$

Quantum Holism

$$L + P \approx L' + P'$$

$\uparrow \quad \uparrow \quad \uparrow \quad \uparrow$
 old physics new physics new physics new physics

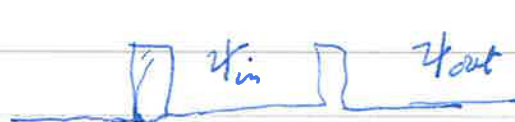
Post, Lucasiewicz 1924.

Zawirski Particle formation - Desbordes
(+ Strauss).

Reichard: Normal system - properties undetermined
phenomenon - observable - measurement
Transformation - what happens between events - phenomena.

A_1, A_2 not meaningless, but is not
a sufficient cause undetermined truth - also.

Barner Distribution Eq



$$\frac{h^2 k^2}{2m} = E$$



$$\langle \frac{p^2}{2m} \rangle \geq 0$$

$$\langle T \rangle + \langle V \rangle = \text{const} = E$$

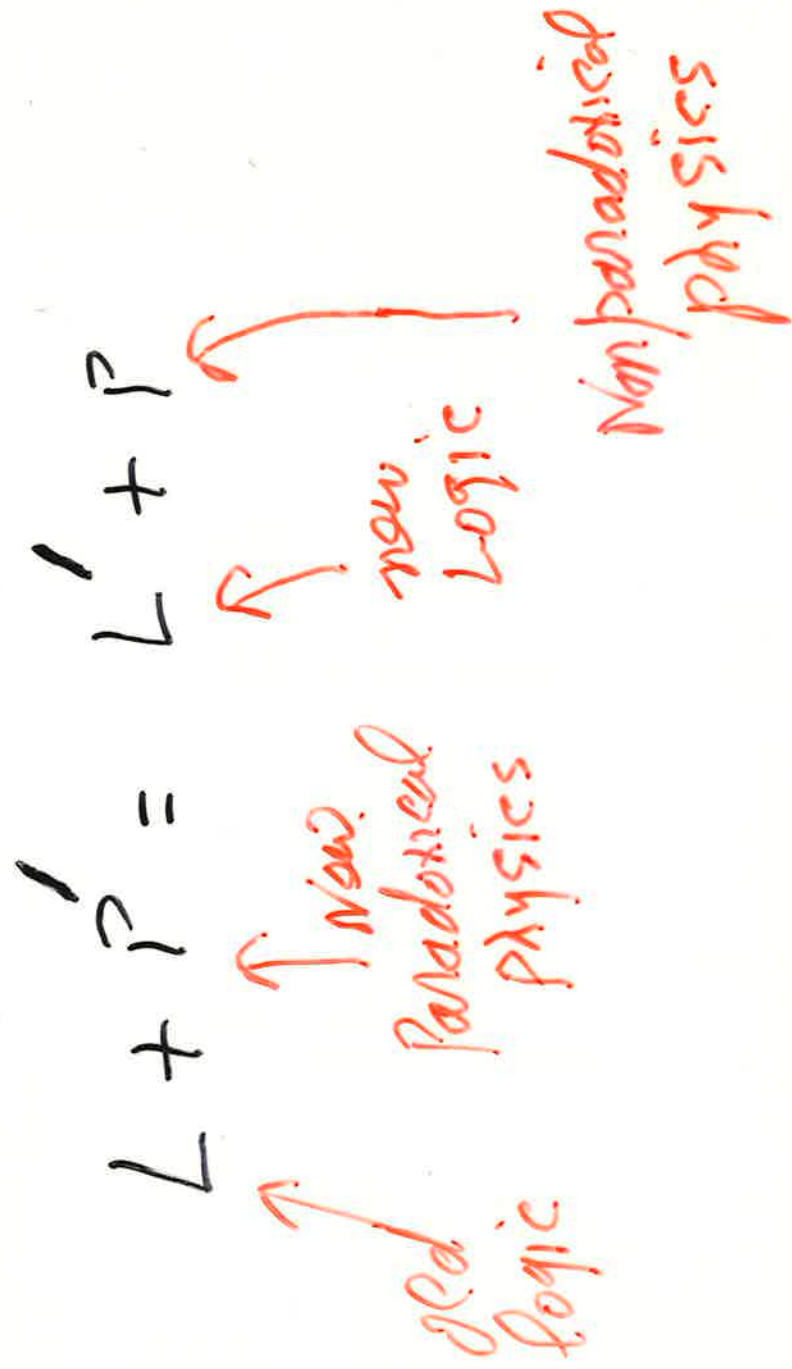
$$\langle T \rangle = E - \langle V \rangle$$

$$\psi_2 = \int_{R_1} \psi(x) |x\rangle dx \quad , \quad \psi_1 = \int_{R_1} \psi(x) |x\rangle dx \quad - \quad , \quad \psi_3 = \int_{R_3} \psi(x) |x\rangle dx$$

$$\langle p | \psi_3 \rangle = \int_{R_3} e^{-ipx} \psi(x) dx$$

$$= \int_{R_3} e^{-i(p-q)x} dx = \frac{1}{p-q} e^{-i(p-q)x}$$

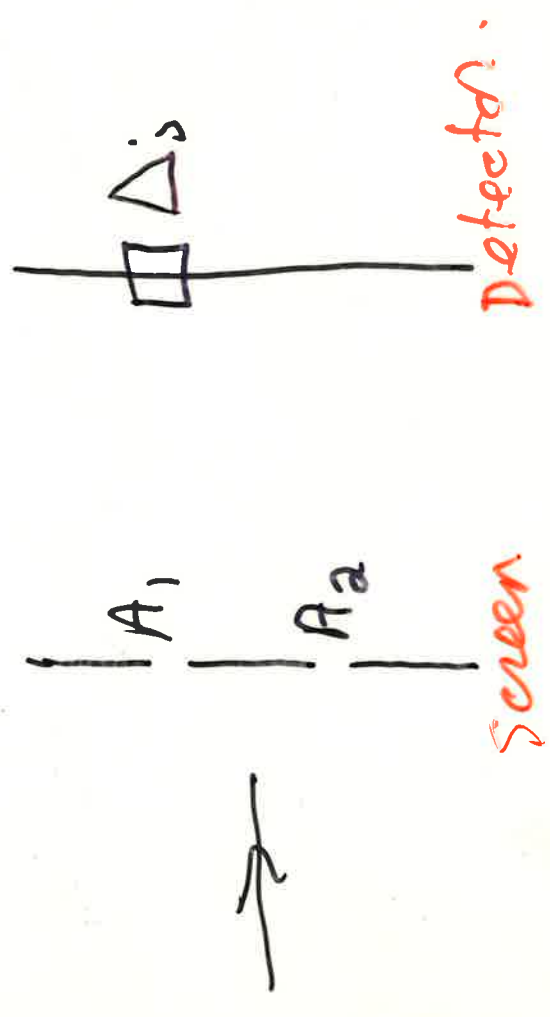
Quinean Holism



The Putnam Proposition

$$\frac{\text{Geometry}}{\text{GR}} = \frac{\text{Logic}}{\text{QM}}$$

PUTNAM AND THE 2-SLIT EXPERIMENT



R_i is proposition that electron hits detector in cell Δ_i , where the associated projector P_i is defined by $P_i = \int_{x \in \Delta_i} dP(x)$

A_1 and A_2 are propositions corresponding to projectors onto the time-evolution ψ_1 and ψ_2 of states associated with passage through one or other slit

(12)

We shall also use $\mathcal{P}\phi$ to denote the normalized state vector

$\frac{\mathcal{P}\phi}{\|\mathcal{P}\phi\|}$ in appropriate

contexts.

②

Then

$$\text{Prob}(R_i | A_1 \vee A_2) = \frac{\text{Prob}(R_i \wedge (A_1 \vee A_2))}{\text{Prob}(A_1 \vee A_2)}$$

$$= \frac{\text{Prob}((R_i \wedge A_1) \vee (R_i \wedge A_2))}{\text{Prob}(A_1 \vee A_2)}$$

Using distributive law

$$= \text{Prob}(R_i | A_1) \times \frac{\text{Prob}(A_1)}{\text{Prob}(A_1 \vee A_2)}$$

$$+ \text{Prob}(R_i | A_2) \times \frac{\text{Prob}(A_2)}{\text{Prob}(A_1 \vee A_2)}$$

$$= \frac{1}{2} \text{Prob}(R_i | A_1) + \frac{1}{2} \text{Prob}(R_i | A_2)$$

= classical pattern with no

interference

Buttram blocks this request by denying the distributive law

But, in fact, the law is trivially valid since $P_i \wedge P_{\neg i} = P_i \wedge P_{\neg i} = P_i \wedge (P_{\neg i} + P_{\neg i}) = 0$

For a 1-dimensional projector P

interpret

$$\begin{aligned} \text{Prob}_\phi(F|P) &= \text{Prob}_\phi(F|P\phi) \\ &= \text{Prob}_{P\phi}(F) \end{aligned} \quad (1)$$

For the 2-slot experiment we need to evaluate

$$\text{Prob}_\phi(R_i | P_1 + P_2) \quad (2)$$

F-P note that

$$P\phi \Rightarrow (P_1 + P_2)\phi \Leftrightarrow (P_1 + P_2)\phi \quad (3)$$

where material equivalence $\Leftrightarrow$ is defined by

$$A \Leftrightarrow B \text{ if } (A \cap B) \vee (\sim A \wedge \sim B)$$

F-P then substitute $(P_1 + P_2)\phi$ for $P_1 + P_2$ in (2) to obtain the correct Lüders result

$$\text{Prob}_\phi(R_i | P_1 + P_2) = \text{Prob}_\phi(R_i | (P_1 + P_2)\phi) = \text{Prob}_{(P_1 + P_2)\phi}(R_i) \quad (4)$$

④

In classical logic

$A \leftrightarrow B$ as defined above
can be rewritten as

$$(A \supset B) \wedge (B \supset A)$$

with $A \supset B = \neg A \vee B$

This satisfies the distributive law, which fails in QL , but Hellman (1981) showed

$$A \leftrightarrow B = (A \supset B) \wedge (B \supset A)$$

where $A \supset B = \neg A \vee (A \wedge B)$
defines the Sasaki horseshoe which reduces to material implication if we are allowed to distribute.

But note, Deduction Theorem fails

we cannot claim $A \supset B$ is a
 $A \Rightarrow B$ iff tautology.

⑤

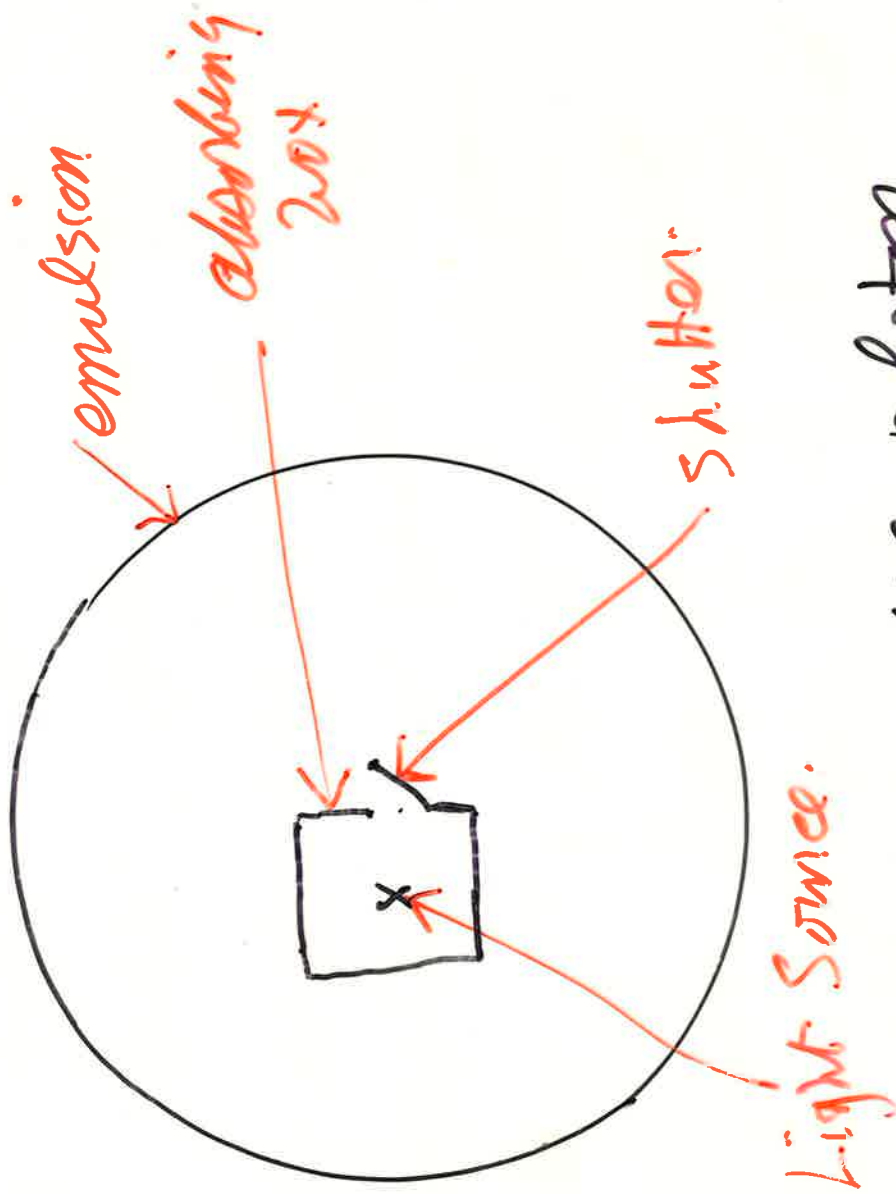
Note Conditionalization
should really be analysed
as a subjunctive conditional
with a probabilistic consequent
i.e. we should interpret $\text{Prob}(A/B)$
 $= P$ as

$$B \Rightarrow (\text{Prob}(A) = P)$$

But then we cannot substitute
equivalent propositions for B ,
even in the classical propositions
since any two false propositions
are materially equivalent.
However, this move is licensed
if the material equivalence is
provable from some background
assumption. still true if the equivalence
? Is this still true if the equivalence
is only provable quantum-
logically.

⑥

The PUTNAM THOUGHT EXPERIMENT



At time t at which photon reaches the detector, the state is

$$\Psi = \frac{1}{\sqrt{2}} (\Psi_{in} + \Psi_{out})$$

where $\Psi_{out} = (\sum_i p_i^{out}) \Psi$

$$\Psi_{in} = (\sum_i p_i^{in}) \Psi$$

p_i^{out} projects onto the i^{th} cell outside the box
 p_i^{in} - - - - - inside the box

$$\textcircled{7} \quad \text{So } \sum_i p_i^{\text{in}} + \sum_i p_i^{\text{out}} = 1$$

Now suppose we measure p_A^{out} , which projects onto Δ_A which encloses a small region of emission, and find the value!

Then, says Putnam, we know the value (viz.) of total p_A^{out} and of p_{out} , which

clearly do not commute.

But this is confused:

What we know is the value of p_A^{out} and of the compatible

projector $\sum_i p_i^{\text{out}}$, but knowing

$\sum_i p_i^{\text{out}}$ (i.e. that the photon is out of

the box) is quite different from knowing $p_{\text{out}} = (\sum_i p_i^{\text{out}}) \mathbb{I}$

⑧

The only way we could
claim to know the
cotton would be to invade

$$P_T \Rightarrow \left(\sum_i P_i^{\text{out}} \leftrightarrow \sum_i P_i^{\text{out}} \right) F$$

i.e. the Friedman-Putnam
move.

Probability and Quantum Mechanics

Some Problems

Thursday 4-4-98

1. A problem for Putnam
2. A problem for Fine
3. A problem for Stapp.
4. A problem for Cartwright

1. Best argument in Logic, Quantum and the 2-slit experiment.

$$2. \text{Loc} \rightarrow \text{Bell} \quad \therefore \neg \text{Bell} \rightarrow \neg \text{Loc.}$$

But says Fine $\text{Bell} \rightarrow \text{J.D.}$

joint distributions for incompatible observables

So $\text{J.D.} + \text{Loc} \rightarrow \text{Bell}$

$$\therefore \neg \text{Bell} \rightarrow (\neg \text{Loc}) \vee \neg \text{J.D.}$$

But Fine's theorem says.

$$\exists R(B(R)) \text{ Bell} \rightarrow \exists P(J(P))$$

↓ probability space ↓ state for J.D.

$$\rightarrow \exists R(J(R)) \quad (\text{Wald})$$

↓ self model. of P.

Now Svetlana Redhead Brown & Butterfield (1988) showed

$$\exists R' [(\sim J(R')) \wedge B(R')] \quad \rightarrow \text{Bell inequality}$$

Fine argued

$$\exists R' (\sim J(R')) \rightarrow \sim B(R')$$

So SBBV had proved $\exists R' (\sim B(R') \wedge B(R'))$
i.e. contradiction?

But all Fine should have concluded

$$\sim \exists R' (J(R')) \rightarrow \sim B(R') \\ \vee R' (\sim B(R')) \\ \rightarrow \sim B(R')$$

$$\sim \exists R (J(R)) \rightarrow \vee R (\sim B(R)) \\ \rightarrow \sim B(R')$$

3. PLCD : The result of a measurement on one particle which could be performed does not depend on the setting of a distant piece of apparatus.

The strict correlation argument for determinism
The Elby objection to the strict correlation
for argument for determinism.

4.

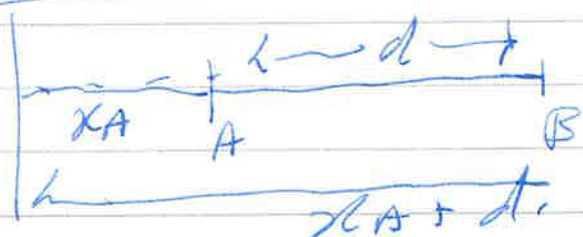
Prob⁴ (a | 2)partial
counts

move to Prob (a | 2 n 4)

In classical physics

Determine state space from values of physical magnitudes. (n.v. functions on state space)

such example

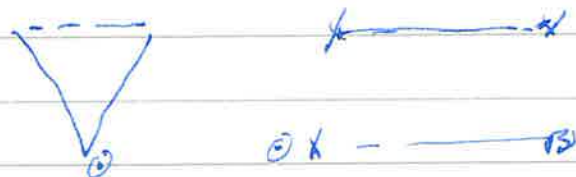
CM.State: Collection of all values of dynamical
and relations between dynamical

$$\text{So } E = P^2/2m \text{ etc.}$$

$$x_B = x_A + d.$$

QMstate: collection of all probs for observables
at consistent probs.

or. prob(a), prob(b), prob(a|b)

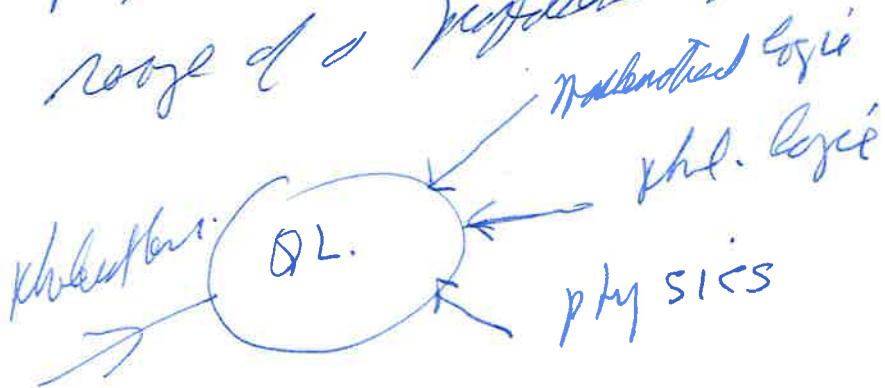


Transparencies

St. Andrews

- Show
1. Inputs to QL
 2. Reformulation & sketching
 3. Historical development of QL
 4. context
 18. Quantum states & Putnam's states
 14. Putnam's d. logic
 15. Putnam's to Paradox.

15a. New Transparency.
Proposition about location of states
Propositions don't refer to bundles
Proposition operators having value
range of a proposed operator



Logic, Quanta and the 2-jet Experiment

Order of Transparencies

- 1 Inputs to QL
- 3 & 4 Historical Development of QL.
- 14 Putnam & QL.
- 1a 6 series of P
- 18 Quantum states & Putnam states
- 15 Putnam and the Paradoxes.
- 1 Putnam and the 2-jet exp.
- 2 contd
- 12 Conversion for P & F
- 3 Friedman & Putnam (1978)
- 4 Defn Material equivalence contd.
- 5 Conditionally dec.
- 6 Putnam's target exp. (1981)
- 7 contd
- 8 Flaws in Putnam's argument.
- 9a The 2-jet version of Putnam's argument
- 9 The Saw contd.
- 10 Joint measurement sequences.

Admissible truth valuations for

Quantum Logic

$Val: \{u\} \rightarrow B_2$ is an admissible valuation iff

A₁ There is a 1-dimensional

subspace N s.t. for every

subspace M , $Val(M) = 1$ iff $N \subseteq M$

A₂ 1. $Val(u) = 1$ iff $Val(2u) = 0$

2. If $Val(n) = 1$, and $N \subseteq M$

then $Val(m) = 1$

3. In any orthonormal basis

$\{ |q_i\rangle \}$ for a maximal

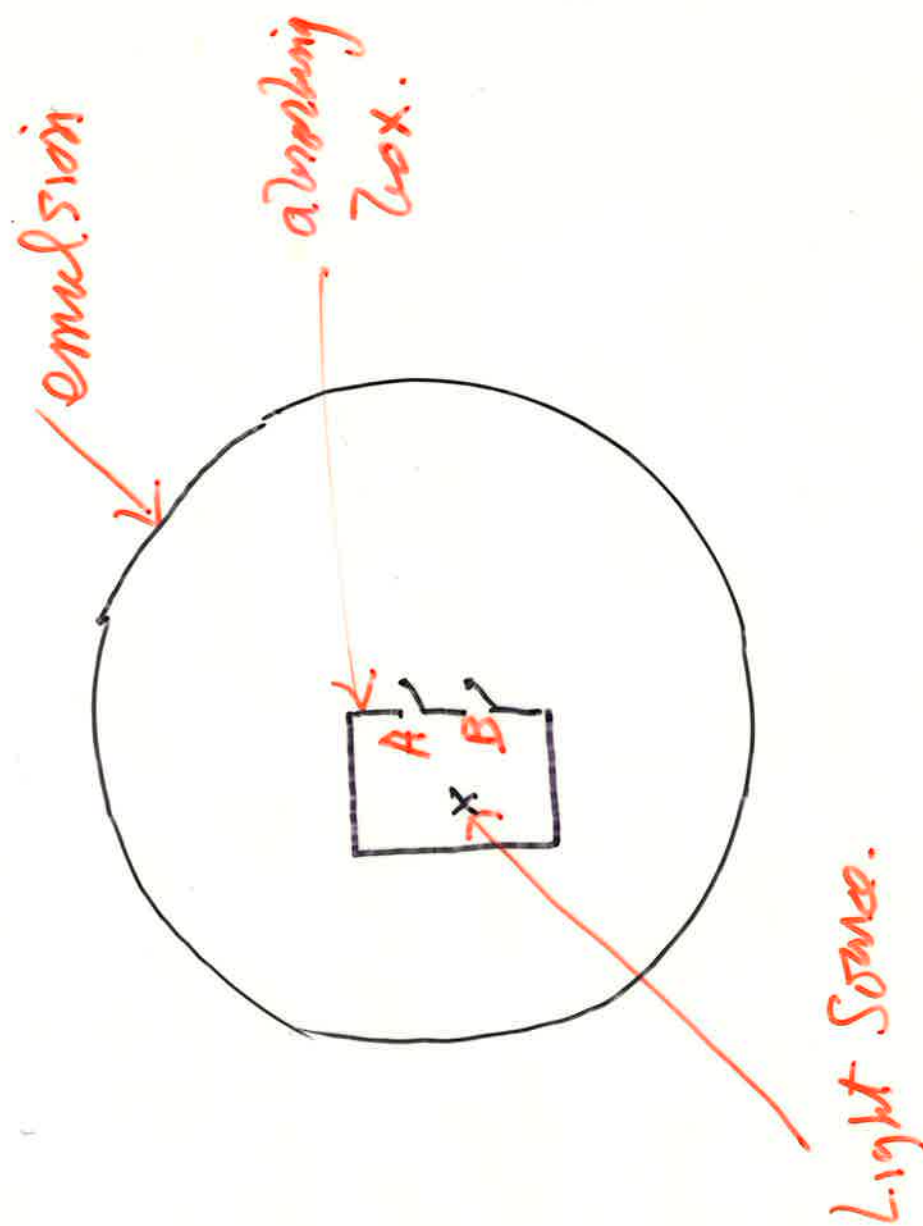
observable Q , $Val(q_i) = 1$ for

some i , and hence from 1. and 2.

$Val(q_i) = 0$, $\forall i \neq i$.

9a

The 2-Slit Version of Futnam's Experiment



The 2-slit version ①

Bohm's Experiment contd

$$\Psi = \frac{1}{\sqrt{2}} \Psi_{in} + \frac{1}{2} \Psi_{out}^A + \frac{1}{2} \Psi_{out}^B$$

where Ψ_{out}^A is state of photon passing
of detection of photon passing
through shutter A similarly
for Ψ_{out}^B and shutter B.

Measuring $P_A = 1$ again
allows us to know the
compatible proposition $\sum_i P_{i,out}$
which says the photon is outside
the box, but not the value
let alone.

$$\sum_i P_{out}^A + P_{out}^B = 1$$

$$P_{out}^A \text{ or } P_{out}^B.$$

Simultaneous Measurement (10)

in QM of Incompatible Observables

$$[Q, P] \neq 0$$

- 1.) Prepare state $|q_i\rangle$ and then measure P , obtaining the value p_i .
 q_i and p_i are claimed to be simultaneous values for Q and P just before P was measured.

- 2.) If $|\phi\rangle$ is not an eigenstate of P or Q measure Q together with a compatible observable X , which in the state $|\phi\rangle$ has the same probability distribution as P .
This is the basis of the "time-of-flight" method for simultaneously measuring position and momentum.

Putnam and the Skolem paradox - Handout.

I. The model theoretic argument

Assumptions behind the argument

(1) Our beliefs (perhaps also our linguistic practices) about the world, or even our scientific theories, can be framed reasonably accurately as a theory Σ cast in a regimented language $\mathcal{L}(\Sigma)$ of the kind mathematical logic normally deals with.

The view about reference is then:

(2) There is a fixed function REF taking each closed term t in $\mathcal{L}(\Sigma)$ onto an item in the language-independent totality that makes up THE WORLD. (REF[C] is the range of this function.)

This amounts to:

(2') THE WORLD (which of course contains all the things in REF[C]) is a fixed interpretation of $\mathcal{L}(\Sigma)$ in the usual model-theoretic sense of interpretation; indeed we can call it the *intended* interpretation of the metaphysical realist speaker of $\mathcal{L}(\Sigma)$.

Moreover:

(3) If f is any homomorphism on THE WORLD to any other structure $f[\text{THE WORLD}]$ (quite possibly with the same domain(s) as THE WORLD), and f is not the identity map, then $f[\text{THE WORLD}]$ is an *unintended* interpretation.

Consequences

The first thing to notice is that:

(i) REF determines REF(True) = the set of all truths about THE WORLD formulable by $\mathcal{L}(\Sigma)$.

Aim of the model theoretic argument

REF(True) does not conversely determine REF without already knowing that we are dealing with THE WORLD.

Thus, we have a circle:

We want REF to point out the elements of THE WORLD. Our main tool in doing this will, we hope, be (part of) REF(True); but we need THE WORLD *together with* REF(True) to pin down REF.

The means of doing this is to show:

There are *unintended* interpretations of $\mathcal{L}(\Sigma)$ in the sense of 'intended' given above, i.e., interpretations other than THE WORLD, which make exactly the same sentences of $\mathcal{L}(\Sigma)$ true as THE WORLD does, REF(True).

The arguments

1. The Löwenheim-Skolem argument.

The *Löwenheim-Skolem Theorem (LST)*. Let $\mathfrak{A}$ be any (infinite) structure which is model of any consistent set of first-order sentences Γ . Given any infinite cardinality different from that of the domain of $\mathfrak{A}$, there is another structure $\mathfrak{B}$ of that cardinality which differs from $\mathfrak{A}$ *only* in the matter of cardinality, and which *also* satisfies the sentences Γ .

Hence, suppose our Σ happens to be true of THE (infinite) WORLD, and thus that its true sentences are exactly those of REF(True). Then the LST says that there are other structures $\mathfrak{B}$ which also satisfy REF(True) and which cannot possibly be THE WORLD.

2. The permutation argument.

Basic theorem: If two structures are *isomorphic*, then they are *elementarily equivalent*.

This is important because we have

Def.: $\mathfrak{A}$ and $\mathfrak{B}$ are *elementarily equivalent* if they satisfy the same sentences of $\mathcal{L}(\Sigma)$.

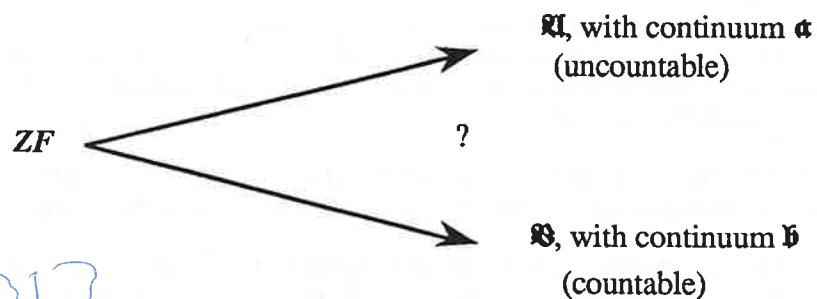
Now assume that THE WORLD is a model of Σ (thus makes REF(True) all true), and let π be a non-trivial one-to-one permutation of the elements in THE WORLD. For each relation R on elements of THE WORLD we can define a new relation R' on the permuted elements by

$$R'(\pi(a_1), \dots, \pi(a_n)) \text{ iff } R(a_1, \dots, a_n)$$

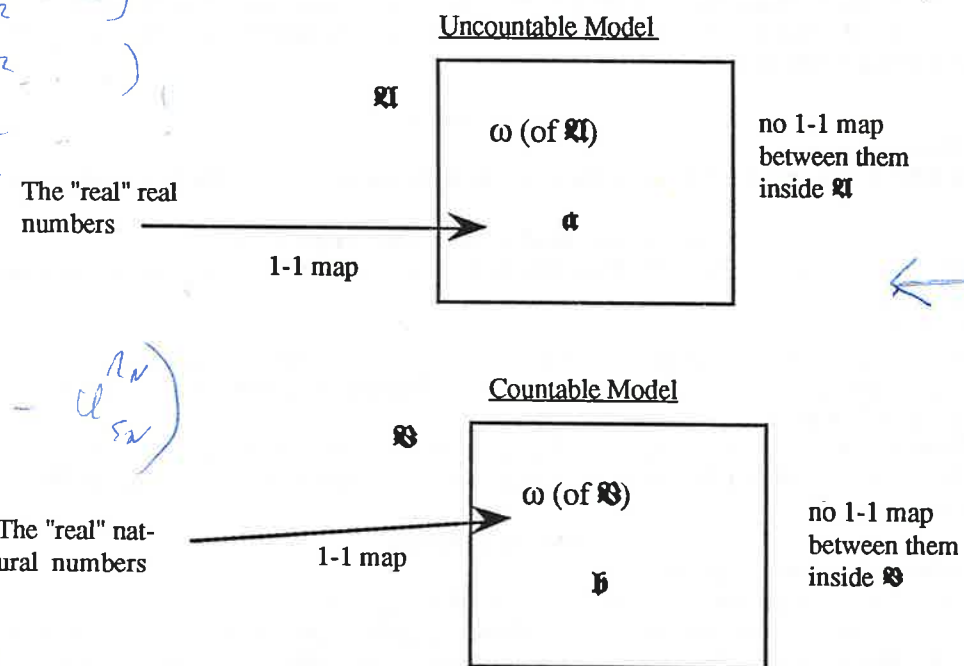
We can make a new model $\mathfrak{A}$ out of these permuted elements and the new relations. This $\mathfrak{A}$ will be isomorphic to THE WORLD. For example, if the sentence $r(t_1, \dots, t_n)$ is made true in THE WORLD by the fact that $R(a_1, \dots, a_n)$ holds, then it will also be made true by the fact about $\mathfrak{A}$ that $R'(\pi(a_1), \dots, \pi(a_n))$. By the basic theorem, then, THE WORLD and $\mathfrak{A}$, being isomorphic are also elementarily equivalent, and $\mathfrak{A}$ also satisfies REF(True).

II The Skolem paradox

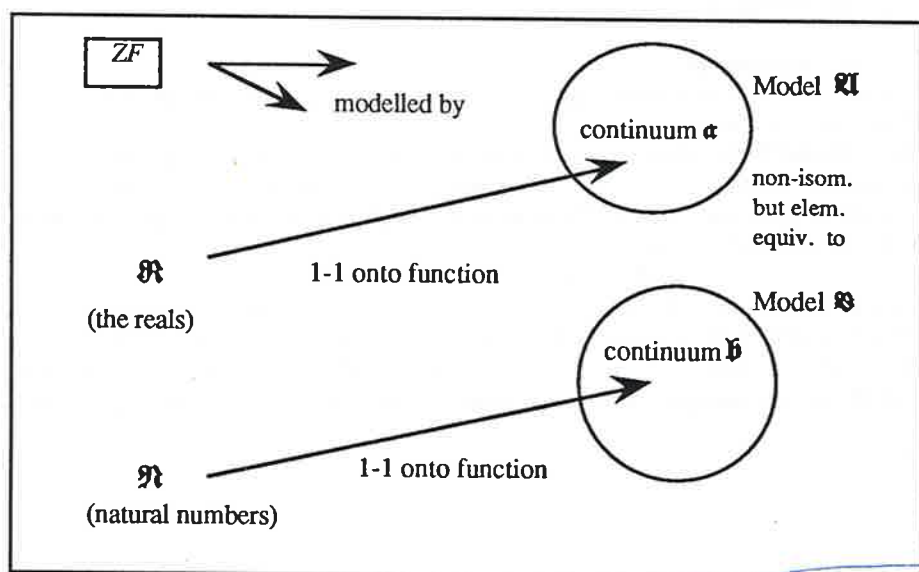
1. The 'paradox'



2. The "inside"/"outside" explanation



3. The "inside"/"outside" explanation properly seen



4. Internal skolem paradox

Central result inside ZF :

If the system ZF is consistent, then the predicate $C(x)$ is not absolute.

Absoluteness

Suppose $u = \{x: \phi(x)\}$. Then:

u is *absolute* if for all φ (satisfying certain minimal conditions) $\{x: \phi(x)\}^\varphi = \{x: \phi(x)\}$.

Correspondingly:

u is *not* absolute if we can find a φ satisfying the minimal conditions such that

$\{x: \phi(x)\}^\varphi \neq \{x: \phi(x)\}$.

In the case that interests us, $P(\omega)$ is not absolute, but ω is. Moreover, the non-absoluteness of $P(\omega)$ amounts to saying that we can find a non-empty transitive φ (satisfying the minimal conditions) such that $\varphi \cap P(\omega)$ is not the same as $P(\omega)$.